Verifying Emergence of Bounded Time Properties in Probabilistic Swarm Systems

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17 July 2018
IJCAI 2018

Robot Swarms

Introduction

- Swarms of drones ("agents") follow certain protocols in order to co-operate and achieve an overall goal.
- These protocols are sometimes probabilistic.
- Often, the goal can only be achieved when a sufficient number of agents is present.
- We call properties that will hold if there are enough agents in the system *emergent properties*.
- Existing research in probabilistic model checking allows us to check finite systems with a fixed number of agents [KDF12].
- Separately, parameterised model checking techniques allow us to check systems with a possibly unbounded number of agents [KL16].

But what about probabilistic systems with a possibly unbounded number of agents?

Agents

- We present a semantics based on a probabilistic modification of [KL15], which in turn is inspired by interleaved interpreted systems [Fag+95].

Definition (Agent Template)

An *agent template* is described by a tuple $T = \langle S, \iota, Act, P, t \rangle$ where:

- S is a non-empty set of states, with $\iota \in S$ is an initial state,
- $Act \neq \emptyset$ is a non-empty set of possible agent actions,
- $P : S \times Act \to [0, 1]$ gives the probability distribution for the agent's next action given its current state,
- $t : S \times S_E \times \mathcal{P}(Act) \times Act \to S$ is the agent's transition function and gives the agent's next state given its current state, the environment's current state, the actions performed by the other agents and the action performed by this agent.

Environment and Swarm System

Definition (Environment)

An *environment* is described by a tuple $E = \langle S_E, \iota_E, t_E \rangle$ where:

- $S_E \neq \emptyset$ is a non-empty set of environment states,
- $\iota_E \in S_E$ is a distinguished initial state,
- $t_E : S_E \times \mathcal{P}(Act) \to S_E$ is the environment's transition function and gives the environment's next state given its current state and the actions performed by the agents.

Swarm System

Definition (Swarm System)

Define a *swarm system* by $\mathcal{S} = \langle T, E, \mathcal{V} \rangle$ where $\mathcal{V} : S \times S_E \rightarrow \mathcal{P}(AP)$ is a labelling function on a set of atomic propositions AP.

- A swarm system is a description of an unbounded number of systems that can be obtained by choosing a different number of agents.
- We use $\mathcal{S}(n)$ to denote the concrete system with n agents.

Probabilistic Bounded Time Properties

Definition (Probabilistic Bounded Time Properties)

We will consider properties from a fragment of PCTL which we will call *probabilistic bounded time properties*. These are expressed by formulas of the form $P_{\geq x}[\psi]$ where ψ is generated by the grammar:

$$\psi ::= X^k \phi \mid \phi \, U^{\leq k} \phi$$

$$\phi ::= \mathsf{true} \mid (a, i) \mid \neg\phi \mid \phi \wedge \phi$$

where $a \in AP$ and $k, i \in \mathbb{N}$.

Example

The formula $P_{\geq 0.5}[X^6(\text{connected}, 1)]$ says "with probability at least 0.5, agent 1 will be connected after 6 time steps."

Emergence

- We want to identify properties of the system that hold when there are enough agents present.

Definition (Emergence)

Given a probabilistic swarm system $\mathcal{S}$, a formula $P_{\geq x}[\psi]$ is said to be an *emergent property* of $\mathcal{S}$ if there is some t such that for all $t' \geq t$ we have $\mathcal{S}(t') \models P_{\geq x}[\psi]$. If this is the case we call t an *emergence threshold*.

Emergent Property Identification

Definition (EPI)

Given a probabilistic swarm system $\mathcal{S}$ and a formula $P_{\geq x}[\psi]$, output *true* if it is an emergent property and *false* if it is not.

Observation

If it is possible with some probability larger than 0 for an action to occur after k time steps, given enough agents it can be made to occur with an arbitrarily high probability $P < 1$.

Abstract Model

Definition (Abstract Model)

Based on this observation, given a probabilistic swarm system $\mathcal{S}$ and some $k, m \in \mathbb{N}$ we define a system $D_{\mathcal{S},k,m}$ which we will refer to as the *abstract model*. This will capture the first k time steps of the behaviour of m agents and an environment that have been interfered with in every possible way by arbitrarily many other agents.

EPI Decision Procedure

Observation

As we consider larger and larger systems, these will behave like $D_{\mathcal{S},k,m}$ for the first k time steps with probability approaching 1.

Theorem

Given a probabilistic swarm system $\mathcal{S}$ and a formula $\phi = P_{\geq x}[\psi]$ then if $D_{\mathcal{S},k,m} \models P_{>x}[\psi]$ we have that ϕ is an emergent property of $\mathcal{S}$. Similarly, if $D_{\mathcal{S},k,m} \models P_{<x}[\psi]$ it is not.

- This theorem gives us a decision procedure for EPI by constructing the abstract model and checking the specification in this.
- There is a small technical detail when the probability is of ψ holding is exactly x, this is addressed in the paper.

Emergence Threshold Identification

Definition (ETI)

Given a probabilistic swarm system $\mathcal{S}$ and a formula $P_{\geq x}[\psi]$, find an emergence threshold for this property if it exists or correctly claim its non-existence.

Observation

A sufficient condition for every possible action at each step $k' \leq k$ to be performed is to have at least one agent follow every possible path of length k.

- We give a procedure for ETI based on the observation above by constructing and solving a numerical equation to give a number of agents large enough.

Evaluation

- We implemented our algorithms into an open-source toolkit `PSV-BD`, based on the probabilistic model checker PRISM [1].
- We used our toolkit to evaluate an *ant coverage* algorithm in which agents must visit every cell in an $n \times n$ grid following a probabilistic algorithm.
- We checked the property that "with probability at least p every cell is visited within k time steps by some agent," expressed by $P_{\geq p}[F^{\leq k}\text{allVisited}]$.

[1] `http://www.prismmodelchecker.org/`

Results

n	p	k	threshold	EPI time (s)	ETI time (s)
3	0.25	7	16	2.1	2.3
3	0.50	7	20	2.1	2.3
3	0.75	7	26	2.1	2.3
3	0.95	7	38	2.1	2.3
3	0.99	7	51	2.1	2.3
5	0.25	13	70	2.2	81
5	0.50	13	85	2.2	81
5	0.75	13	105	2.2	81
5	0.95	13	145	2.2	81
5	0.99	13	183	2.2	81
10	0.25	31	-	3.9	*timeout*

Table: Our results checking $P_{\geq p}[F^{\leq k}\text{allVisited}]$ for different values of n, k and p.

Conclusion

- We have given a semantics to reason about emergence of behaviours in probabilistic swarm systems.
- We have presented decision procedures for the identification of emergent properties and emergence thresholds in these semantics.
- We have implemented our decision procedures and used them on a simple example.
- In future work, we would like to look at more complex protocols.

References

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